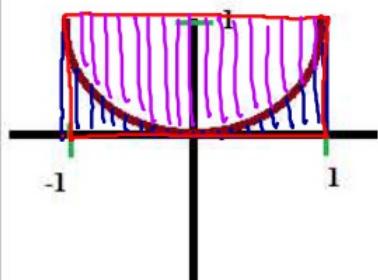


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Find the average value of the function without integrating.

$$16. f(x) = 1 - \sqrt{1 - x^2}$$

Geometry



$$\begin{aligned}\text{Avg Value} &= \frac{\text{Area}}{\text{width}} \\ &= \frac{\int_{-1}^1 f(x) dx}{1 - (-1)}\end{aligned}$$

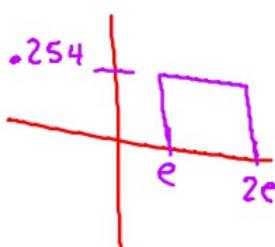
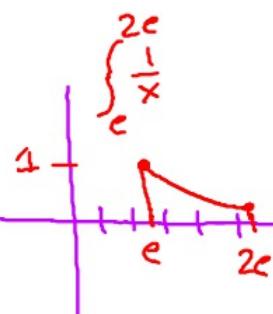
$$\begin{aligned}\text{Area Rectangle} &= 2 \\ \text{Area } \frac{1}{4} \text{ circle} &= \frac{1}{2}\pi(1)^2\end{aligned}$$

$$\begin{aligned}&= \frac{2 - \frac{1}{2}\pi}{2} \\ &= 1 - \frac{\pi}{4}\end{aligned}$$

Find the avg value using antiderivatives

$$32. y = \frac{1}{x} [e, 2e]$$

$$\begin{aligned}\text{Avg Value} &= \frac{\int_e^{2e} \frac{1}{x} dx}{2e - e} = \frac{[\ln x]_e^{2e}}{e} = \frac{\ln(2e) - \ln e}{e} \\ &= \frac{\ln(2e) - 1}{e} \\ &= \frac{\ln 2 + \ln e - 1}{e}\end{aligned}$$



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$$\frac{1}{e} \int_e^{2e} \frac{1}{x} dx = \frac{\ln 2}{e} = .254$$

$$= \sqrt{\frac{\ln 2}{e}}$$